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The manifestly $Sl(2;\mathbb{Z})$-covariant superstring

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Abstract: We present a manifestly $Sl(2;\mathbb{Z})$-covariant action for the type IIB superstring, and prove κ-symmetry for on-shell IIB supergravity backgrounds.

Keywords: Superstrings and Heterotic Strings, String Duality, Supergravity Models.

Contents

1 Introduction

The Type IIB superstring theory has D=10 IIB supergravity as its effective field theory. Until a few years ago, the $Sl(2;\mathbb{R})$ invariance of the latter [1, 2] was thought to be an artefact of the field theory approximation to string theory, but it is now believed that IIB superstring theory is itself an approximation to some underlying non-perturbative theory in which an $Sl(2;\mathbb{Z})$ subgroup of $Sl(2;\mathbb{R})$ survives as a symmetry [3, 4]. To the extent to which this theory can be said to be a string theory it describes an entire $Sl(2;\mathbb{Z})$ orbit of '(p,q)' strings [5, 6] with the (1,0) string being the Green-Schwarz (GS) IIB superstring and the (0,1) string the D-string. This explains why both the usual IIB superstring action, and that of the D-string, break $Sl(2;\mathbb{Z})$ (the action for the 'fundamental' string breaks the full group, while the D-string action, describing $(p,1)$ strings, breaks it to $\mathbb{Z}$). In a recent paper [7], one of the authors presented a new $Sl(2;\mathbb{Z})$-covariant string action that simultaneously describes the entire $Sl(2;\mathbb{Z})$ orbit of (p,q) strings. We say 'covariant' rather than 'invariant' because an $Sl(2;\mathbb{Z})$-transformation of the worldsheet fields must be accompanied by an $Sl(2;\mathbb{Z})$-transformation of the background. The purpose of this paper is to present the supersymmetric generalization of this action in a form that makes the $Sl(2;\mathbb{Z})$ covariance manifest, i.e. the manifestly $Sl(2;\mathbb{Z})$-covariant IIB superstring.

The construction involves establishing the fermionic gauge invariance known as κ-symmetry. It was shown already in [8] that κ-symmetry of the GS IIB superstring implies constraints on the background that are equivalent to the on-shell superspace constraints of IIB supergravity, but this derivation of them obscures their $Sl(2;\mathbb{R})$-invariance. In contrast, κ-symmetry of the new IIB superstring action implies the on-shell IIB supergravity superspace constraints in $Sl(2;\mathbb{R})$-covariant form. Actually, we shall establish here only that these constraints are sufficient for κ-symmetry but the results of [8] guarantee that they are also necessary. We remark that the IIB on-shell constraints have also been shown to be sufficient for κ-symmetry of the D-3-brane [9], but they have not yet

been shown to be necessary (although this is almost certainly the case). Similar results have been found for other IIB D-branes [10, 11, 12] but the $Sl(2;\mathbb{R})$ invariance of the background is again obscured. Thus, the action proposed here offers by far the simplest route to a derivation of the IIB supergravity constraints in manifestly $Sl(2;\mathbb{R})$-covariant form as integrability conditions. Another motivation is the potential insight that a manifestly $Sl(2;\mathbb{Z})$-covariant string might provide into a conjectural 12-dimensional theory underlying IIB superstring theory [13, 14, 15].

We begin with a review of the string action of [7], rewriting it in a form that makes the $Sl(2;\mathbb{Z})$ covariance manifest. The action for the corresponding manifestly $Sl(2;\mathbb{Z})$-covariant IIB superstring is formally identical, but the background is IIB superspace. We prove the κ-symmetry of this action subject to the on-shell constraints of IIB supergravity. We conclude with a brief discussion of some potential applications of our results.

2 The $Sl(2;\mathbb{Z})$-invariant String

The construction of [7] puts together two earlier observations:

(i) The tension of a super p-brane may be generated dynamically in a formulation containing a world-volume p-form potential, which has no local degrees of freedom [16].

(ii) The Born–Infeld (BI) field on the world-sheet of the D-string also has no local degrees of freedom, but the integer quantization of its electric field generates NS-NS charge [17].

These two observations make it natural to replace the D-string tension, equivalent to the magnitude of the RR 2-form charge, by a second BI potential. The two worldsheet gauge fields can be assembled into an $Sl(2)$ doublet, thereby allowing an $Sl(2)$-invariant coupling to the background IIB supergravity fields. The tension of this $Sl(2;\mathbb{Z})$-invariant string is generated dynamically; it depends on both the integer RR and NS-NS charges, both of which arise due to electric field quantization.

Thus, the world-sheet fields comprise not only the target space coordinates but also an $Sl(2;\mathbb{R})$ doublet A_r, $r = 1, 2$, of abelian gauge potentials. The latter enter the action via their 'modified' field-strengths

$$F_r = dA_r - B_r \tag{2.1}$$

where B_r are the pullbacks to the worldvolume of the NS and RR 2-form potentials. We use the same symbol to denote spacetime forms and their pullbacks as it should be clear which is intended from the context. In order to write an $Sl(2;\mathbb{R})$-invariant 'F^2-term', one needs the background scalars. These belong to the coset $Sl(2;\mathbb{R})/SO(2)$ or, equivalently $SU(1,1)/U(1)$. We shall use the latter description here. Thus, the scalars are represented by a complex $SU(1,1)$ doublet $\mathscr{U}^r$ ($r = 1, 2$) satisfying the $SU(1,1)$-invariant constraint

$$\tfrac{i}{2}\varepsilon_{rs}\mathscr{U}^r\bar{\mathscr{U}}^s = 1\,. \tag{2.2}$$

Viewing $\mathscr{U}$ as a 2×2 matrix on which $SU(1,1)$ acts from the left, there is a commuting action of $U(1)$ from the right; we normalize the $U(1)$ charge by taking $\mathscr{U}$ to have unit charge. Gauging this $U(1)$ reduces the number of independent scalars from three to two.

One can move freely between F_r and an $SU(1,1)$-invariant complex field-strength $\mathscr{F}$ using

$$\mathscr{F} = \mathscr{U}^r F_r\,, \qquad F_r = \varepsilon_{rs}\mathrm{Im}\,(\mathscr{U}^s\bar{\mathscr{F}})\,. \tag{2.3}$$

Similarly, $H_r = dB_r$ is the $Sl(2;\mathbb{R})$ doublet of NS and RR background 3-form field strengths, and $\mathscr{H} = \mathscr{U}^r H_r$ the $SU(1,1)$-invariant version. Introducing the left-invariant $SU(1,1)$ Maurer–Cartan forms

$$P = \tfrac{1}{2}\varepsilon_{rs}d\mathscr{U}^r\mathscr{U}^s\,, \qquad Q = \tfrac{1}{2}\varepsilon_{rs}d\mathscr{U}^r\bar{\mathscr{U}}^s\,, \tag{2.4}$$

we can write the Bianchi identity for $\mathscr{H}$ as

$$D\mathscr{H} + i\bar{\mathscr{H}}\wedge P = 0\,, \tag{2.5}$$

where D is the covariant exterior derivative constructed from the $U(1)$ connection Q, which as a consequence of (2.2) is real.

The complex dilaton-axion field of the IIB supergravity background can be constructed as the projective invariant $\mathscr{U}^2(\mathscr{U}^1)^{-1} = \chi + ie^{-\phi}$. It is sometimes convenient to use the $U(1)$ gauge $\mathrm{Im}\,\mathscr{U}^1 = 0$ and identify $\mathscr{U}^1 = e^{\frac{\phi}{2}}$, $\mathscr{U}^2 = e^{\frac{\phi}{2}}\chi + ie^{-\frac{\phi}{2}}$. Since we want to maintain manifest $Sl(2)$-covariance, we will most of the time use the $\mathscr{U}$'s.

To complete the assembly of ingredients needed for the construction of a manifestly $SU(1,1)$-invariant action we define the complex scalar density

$$\Phi = \tfrac{1}{2}\varepsilon^{ij}\mathscr{F}_{ij}\,. \tag{2.6}$$

The $Sl(2)$-invariant string action is

$$S = \tfrac{1}{2}\int d^2\sigma\,\lambda(g + \Phi\bar{\Phi})\,. \tag{2.7}$$

The metric is understood to be the pullback of the $Sl(2;\mathbb{R})$-invariant Einstein metric, and g is its determinant, so the action is manifestly $Sl(2;\mathbb{R})$-invariant or, rather, covariant since the invariance of Φ requires an $Sl(2;\mathbb{R})$ transformation of the background. Note the absence of a Wess–Zumino term; Φ already contains couplings to both the NS-NS and RR sectors. The action is also invariant under space-time scale transformations:

$$X\to\rho X\,, \quad A\to\rho^2 A\,, \quad \lambda\to\rho^{-4}\lambda\,.$$

We will briefly analyze the action (2.7) before moving on to the supersymmetric case (for a complete demonstration that the equations of motion implied by (2.7) are those of a (p,q) string we refer to ref. [7]). The equation of motion for the Lagrange multiplier λ enforces the constraint

$$g + \Phi\bar{\Phi} = 0\,. \tag{2.8}$$

The equations of motion for the A_r read

$$d\left\{\lambda\,\mathrm{Re}\,(\mathscr{U}^r\bar{\Phi})\right\}=0\,. \tag{2.9}$$

The entities inside the curly brackets must take some constant values. These are best understood in a canonical framework. The space-components of the electric fields, the conjugate momenta to A_r, are

$$E^r=\lambda\,\mathrm{Re}\,(\mathscr{U}^r\bar{\Phi})\,. \tag{2.10}$$

When the world-sheet is a cylinder, the values of E^r are quantized to be integers by demanding gauge invariance [6]. We therefore let $E^r=\varepsilon^{rs}n_s$, where n_r is a doublet of integers. This breaks $Sl(2;\mathbb{R})$ to the subgroup $Sl(2;\mathbb{Z})$. Equation (2.10) is readily solved by setting

$$\lambda\Phi=-i\,\mathscr{U}^r n_r\,. \tag{2.11}$$

In order to identify the tension, we continue the canonical analysis to the coordinate sector. The momenta are

$$P_m=\lambda\Big(\dot{X}_m(X')^2-X'_m(\dot{X}\cdot X')\Big)-E^r(B_r)_{mn}X'^n\,, \tag{2.12}$$

from which we deduce the following constraints:

$$\begin{aligned}0&=P\cdot X'\,,\\ 0&=(P_m+E^r(B_r)_{mn}X'^n)^2+(\lambda\sqrt{-g}\,X')^2\,.\end{aligned} \tag{2.13}$$

The effective tension is read off from the second of these equations as

$$T=<\lambda\sqrt{-g}>=<|\mathscr{U}^r n_r|>\ \ . \tag{2.14}$$

It is also clear from (2.13) that E^r are the charges with which the string couples to the background 2-forms B_r. Since the metric entering the action was the $Sl(2;\mathbb{R})$-invariant Einstein metric, this $Sl(2;\mathbb{Z})$-invariant tension refers to the Einstein frame. For constant dilaton and axion, and with $n_r=(p,q)$, the tension becomes

$$T=\sqrt{e^{-\phi}q^2+e^{\phi}(p+q\chi)^2}\,. \tag{2.15}$$

Rescaling to the string frame by $(g_{\mathrm{string}})_{mn}=e^{\frac{\phi}{2}}(g_{\mathrm{Einstein}})_{mn}$, and identifying the string coupling constant as $g_s=e^{\phi}$, the string frame tension takes the well-known form [5]

$$T_s=\sqrt{\left(\frac{q}{g_s}\right)^2+(p+q\chi)^2}\,. \tag{2.16}$$

Note that the tension and the phase of Φ naturally fit together in a 'complex tension' $\lambda\Phi$.

3 The $Sl(2;\mathbb{Z})$-covariant Superstring

The action for the manifestly $Sl(2;\mathbb{Z})$ superstring is formally identical to the bosonic action (2.7), but the 'coordinate' worldsheet fields are now the superspace coordinates $Z^M = (X^m, \theta^\mu, \bar\theta^{\bar\mu})$. Note that the two D=10 chiral spinors of IIB superspace have been assembled into a single complex chiral spinor. As usual, we introduce the frame 1-forms $E^A = (E^a, E^\alpha, E^{\bar\alpha})$ and define the induced worldsheet metric via the pullback of E^a:

$$g_{ij} = E_i{}^a E_j{}^b \eta_{ab}\,, \tag{3.1}$$

where $E_i{}^a = \partial_i Z^M E_M{}^a$. The 2-form field strengths F_r are as before but the 2-form gauge potentials B_r are now pulled back from 2-forms on superspace. The complex 3-form field strength $\mathscr{H} = \mathscr{U}^r dB_r$ is the $SU(1,1)$-invariant superspace field strength introduced in [2]. We now turn to the issue of κ-symmetry.

Consider a local fermionic transformation of the type

$$\delta Z^M = \zeta^\alpha E_\alpha{}^M + \bar\zeta^{\bar\alpha} E_{\bar\alpha}{}^M\,. \tag{3.2}$$

The induced variation of the pullback of a superspace form Ω is given by

$$\delta\Omega = \mathscr{L}_\zeta \Omega = (i_\zeta d + d i_\zeta)\Omega\,. \tag{3.3}$$

We use this to calculate that the variation of the $SU(1,1)$-invariant field-strength $\mathscr{F}$ is

$$\delta\mathscr{F} = -i_\zeta \mathscr{H} - i\bar{\mathscr{F}} i_\zeta P + i\mathscr{F} i_\zeta Q\,, \tag{3.4}$$

where we have used $\delta A_r = i_\zeta B_r$. The variation of the induced metric is

$$\delta g_{ij} = 2E_{(i}{}^a E_{j)}{}^B \zeta^\alpha T_{B\alpha}{}^b \eta_{ab} + c.c. \tag{3.5}$$

where $T_{BC}{}^A$ is the superspace torsion.

At this point we shall use the known on-shell superspace constraints of IIB supergravity. They are [2]

$$\begin{aligned}
&\mathscr{H}_{a\alpha\beta} = 2i(\gamma_a)_{\alpha\beta}\,,\\
&\mathscr{H}_{ab\bar\alpha} = -i(\gamma_{ab}P)_\alpha\,,\\
&T_{\alpha\bar\beta}{}^a = i(\gamma^a)_{\alpha\beta}\,,\\
&T_{\bar\alpha\bar\beta}{}^\gamma = i\delta_{(\alpha}{}^\gamma P_{\beta)} - \tfrac{i}{2}(\gamma_a)_{\alpha\beta}(\gamma^a P)^\gamma\,,\\
&P_{\bar\alpha} = 0\,, \qquad Q_\alpha = 0\,,
\end{aligned} \tag{3.6}$$

where P in the expressions for the dimension $1/2$ parts of $\mathscr{H}$ and T is understood as the spinor component P_α. These are the non-vanishing fields at dimensions 0 and $1/2$, up to components that are obtained from these by complex conjugation. It is straightforward to use these supergravity constraints to obtain the transformation of the constraint ((2.8)): At dimension 0 (which is all that is relevant for bosonic backgrounds) we find

$$\begin{aligned}
\Big\{\delta(g+\Phi\bar\Phi)\Big\}_{(0)} &= 2i\bar E_i \Big\{g\gamma^i\zeta + \Phi\varepsilon^{ij}\gamma_j\bar\zeta\Big\} + c.c.\\
&= -2i\bar E_i\gamma^i\Xi\Big\{\Xi\zeta - \Phi\bar\zeta\Big\} + c.c.
\end{aligned} \tag{3.7}$$

where, as in [16], we have introduced the (real) matrix

$$\Xi = \frac{1}{2}\varepsilon^{ij}\gamma_{ij} \tag{3.8}$$

satisfying

$$\{\Xi, \gamma^i\} = 0 \qquad \Xi^2 = -g\,. \tag{3.9}$$

At dimension 1/2 (at which we find terms involving background fermions) the metric has no variation, while

$$\left\{\delta\bar{\Phi}\right\}_{(1/2)} = -i\bar{P}_\alpha(\Xi\zeta - \Phi\bar{\zeta})^\alpha\,. \tag{3.10}$$

Thus, the full variation of the constraint, in arbitrary (on-shell) backgrounds, is

$$\delta(g + \Phi\bar{\Phi}) = -i(2E_i\gamma^i\Xi + \Phi\bar{P})(\Xi\zeta - \Phi\bar{\zeta}) + c.c. \tag{3.11}$$

If the action is to be invariant the transformation (3.11) must be cancelled by a variation of the Lagrange multiplier λ. This means that the expression in (3.11) must vanish modulo the constraint, which requires ζ to take the form

$$\zeta = \Xi\kappa + \Phi\bar{\kappa}\,, \tag{3.12}$$

where the complex chiral spacetime-spinor parameter κ is arbitrary. Given this, we find that

$$\delta(g + \Phi\bar{\Phi}) = (g + \Phi\bar{\Phi})(2iE_i\gamma^i\Xi\kappa + i\Phi\bar{P}\kappa) + c.c. \tag{3.13}$$

This can clearly be cancelled by a variation of λ.

To summarize, the action is invariant under the following fermionic gauge transformations of the worldsheet fields

$$\begin{aligned}
\delta\lambda &= -i(2E_i\gamma^i\Xi + \Phi\bar{P})\kappa + c.c. \\
\delta Z^M &= (\Xi\kappa + \Phi\bar{\kappa})^\alpha E_\alpha{}^M + (\Xi\bar{\kappa} + \bar{\Phi}\kappa)^{\bar{\alpha}} E_{\bar{\alpha}}{}^M \\
\delta(A_r)_i &= E_i{}^A(\Xi\kappa + \Phi\bar{\kappa})^\alpha(B_r)_{\alpha A} + c.c.
\end{aligned} \tag{3.14}$$

It can be shown that these κ-symmetry transformations effectively remove half of the fermionic degrees of freedom, so that there is an on-shell matching of bosonic and fermionic world-sheet fields. This will be demonstrated below for non-vanishing Φ.

4 Discussion

The action presented in this paper contains in its spectrum the entire orbit of superstrings coupling with charges (p, q) to the RR and NS-NS 2-form potentials. The spectrum is not confined to coprime pairs of charges, but a restriction to this irreducible orbit is clearly consistent, at least at the first-quantized level. Such a restriction preserves the $Sl(2;\mathbb{Z})$ invariance but breaks the scale-invariance. The only sector preserving scale-invariance is the $Sl(2;\mathbb{Z})$ singlet $(p, q) = (0, 0)$. This sector describes a tensionless, or null, superstring, unrelated to the fundamental superstring by any known duality. It too is removed from

the spectrum by the restriction to the single $Sl(2;\mathbb{Z})$ orbit containing the fundamental string. In this case we may assume that Φ is non-zero, and this allows a reformulation of the κ-symmetry transformations.

In section 3, the complex chiral spinor parameter κ was unconstrained but entered into all transformations multiplied by a matrix of half maximal rank. For non-zero Φ the constraint imposed by λ ensures that the induced metric is non-degenerate, so we can divide by $\sqrt{-g}$. Setting $\Phi = e^{i\vartheta}|\Phi|$, we can then rewrite (3.12) as

$$\begin{bmatrix} \zeta \\ \bar{\zeta} \end{bmatrix} = \tfrac{1}{2}(\mathbf{1} + \Gamma) \begin{bmatrix} \zeta \\ \bar{\zeta} \end{bmatrix} , \tag{4.1}$$

where

$$\Gamma = \frac{1}{\sqrt{-g}} \begin{bmatrix} 0 & e^{i\vartheta}\Xi \\ e^{-i\vartheta}\Xi & 0 \end{bmatrix} . \tag{4.2}$$

Note that $\Gamma^2 = \mathbf{1}$, so that the matrix $\frac{1}{2}(\mathbf{1} + \Gamma)$ is a projector. Since $\mathrm{tr}(\Gamma) = 0$ it projects onto a space of half the maximum dimension.

The (p, q) strings treated here represent only part of the BPS spectrum of the non-perturbative IIB superstring theory. One would like a manifestly $Sl(2;\mathbb{Z})$-covariant action for all IIB p-branes. The first case to consider after strings is the D-3-brane, which is an S-duality singlet. The bosonic action was shown in [18, 19] to be $Sl(2;\mathbb{Z})$-covariant in the sense that an $Sl(2;\mathbb{Z})$ transformation of the background can be compensated by a world-volume duality transformation of the BI field strength. A *manifestly* $Sl(2;\mathbb{Z})$-covariant formulation of the linearized bosonic action has been given in [20], and elaborated on in [21, 22]; the essential ingredient is a self-duality condition on a complex BI field. The next case to consider is the S-duality doublet of solitonic and Dirichlet 5-branes. Given that there also exist bound states describing (p, q) 5-branes one would expect there to be an $Sl(2;\mathbb{Z})$-covariant 5-brane action, but there is at present little indication of how this might be constructed.

Finally, it seems possible that one could take the new superstring action presented here as the starting point for an $Sl(2;\mathbb{Z})$-covariant string perturbation theory. Note that although all (p, q) strings other than the (1,0) string are non-perturbative within conventional string theory, the $Sl(2;\mathbb{Z})$ symmetry is not intrinsically non-perturbative because (in contrast to the analogous $Sl(2;\mathbb{Z})$ symmetry of D=4 N=4 supersymmetric gauge theories) it does not exchange electric with magnetic degrees of freedom. Moreover, a generalization of the Veneziano amplitude has been proposed in which poles correspond to states of (p, q) strings [23]. It would be very interesting to investigate whether an $Sl(2;\mathbb{Z})$-covariant perturbation theory based on the action presented here could reproduce this or a similar result. The first step towards such a perturbation theory would seem to be gauge fixing of the κ-symmetry. This can probably be done without breaking manifest Lorentz covariance, along the lines of ref. [24], but it is harder to see how the $Sl(2;\mathbb{Z})$ covariance could also be maintained. It might be made possible through the introduction of $SU(1, 1)$ harmonics. We leave this to the future.

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